Collisions in MD5

... and how to use them for fun and profit.

Mar. 10, 2009

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Outline

1. **Hash functions and their uses**
   - What is a hash function ?
   - The Merkle-Damgård construction
   - Message-Digest algorithm 5 (MD5)
2. **Differential cryptanalysis of MD5**
   - Wang's differential path
   - Deriving a sufficient conditions set
   - Building the collision
3. **Conclusion**

What is a hash function ?

Hash function

Let Σ, Ω be two finite alphabets and n a positive integer. A hash function f is a map :

$$f : \Sigma^* \longrightarrow \Omega^n$$

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3. Second preimage resistant (given $\omega_1 \in \Sigma^*$, it is hard to find $\omega_2 \neq \omega_1$ such that $f(\omega_1) = f(\omega_2)$)
4. Collision resistant (it is hard to find $\omega_1, \omega_2 \in \Sigma^*$ such that $f(\omega_1) = f(\omega_2)$)

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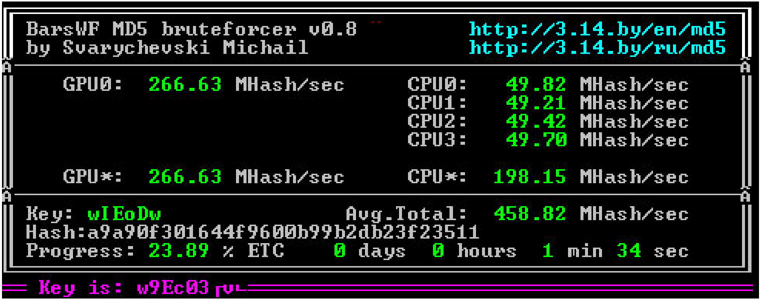
- Brute force can be effective ! (up to 1 billion hashes per second on a desktop PC)
- MD5 : $\Omega = \{0, 1\}$, $n = 128$ is too low for current processing power.



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- Random collisions, not very significant.
- But we want collisions on *meaningful* data, $< 2^{64}$ calls to the MD5.

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Goals

- Compress input : variable length $\rightarrow$ fixed length

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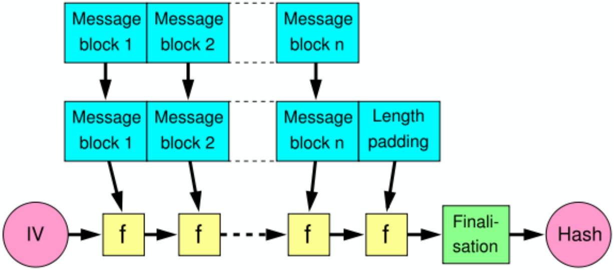

The Merkle-Damgård construction

Construction

- $f : \{0,1\}^n \times \{0,1\}^m \longrightarrow \{0,1\}^n$ is the compression function.
- m is the block size, n the digest size
- IV is a fixed initialization vector
- Length padding is critical for the security of the construction

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Properties

- Proven strength : f fix-start collision resistant and fix-start preimage resistant implies cryptographic strength
- Convenient : single function for the whole process
- Can wreak havoc if compression function has collisions

Message-Digest algorithm 5 (MD5)

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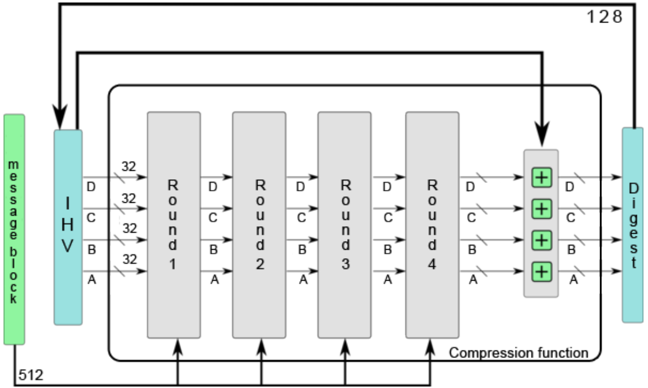
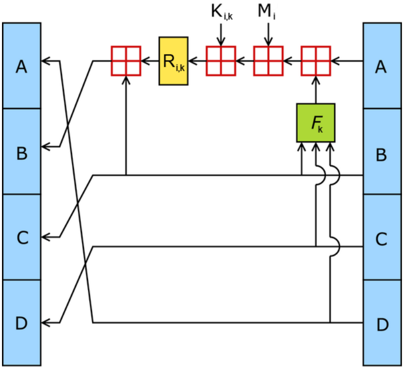


A round of MD5

Description

- One different non-linear function $F_{k \in [1,4]}$ per round
- 16 operations per round on 32-bit slices $M_{i \in [1,16]}$ of the 512 bit input block.
- A constant $K_{i,k}$ is added at each round and a left bit rotation $R_{i,k}$ is applied



Algorithm parameters

Non-linear function

$$F_1(X,Y,Z) = (X \wedge Y) \vee (\neg X \wedge Z)$$
$$F_2(X,Y,Z) = (X \wedge Z) \vee (Y \wedge \neg Z)$$
$$F_3(X,Y,Z) = X \oplus Y \oplus Z$$
$$F_4(X,Y,Z) = Y \oplus (X \vee \neg Z)$$

$K_{i,k}$ "nothing up my sleeve" constants

$$K_{i,k} = \lfloor 2^{32} |\sin(4 * (k-1) + i)| \rfloor$$

Initialization vector

$$\begin{aligned} A_0 &= \texttt{0x67452301} \\ B_0 &= \texttt{0xEFCDAB89} \\ C_0 &= \texttt{0x98BADCFE} \\ D_0 &= \texttt{0x10325476} \end{aligned}$$

Differential cryptanalysis of MD5

Differential Cryptanalysis

- Familly of cryptanalysis methods
- Known as early as 1974 by the NSA, published 15 years later !
- Explore how variations in the input translate to the output, "tickle attack"

Differential path and collisions

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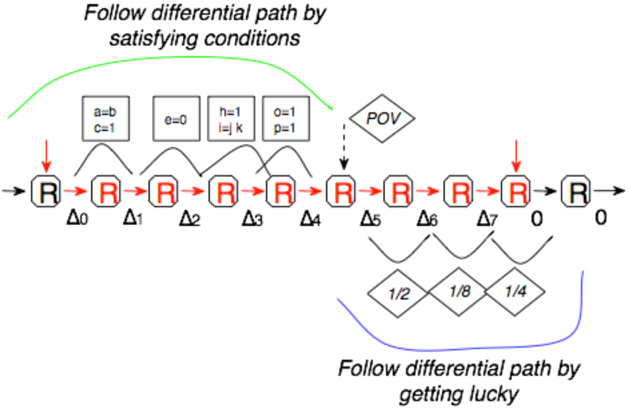
- This sequence of differentials is a roadmap to find the collision
- ... but they’re hard to find

Exploiting a differential path



Multi-bloc differential path

Wang et al. differential path construction

We consider a more general problem : find $(M_0, M_0'), (M_1, M_1')$ such that we have for any IHV_k :

$$
\begin{array}{ccccccccc}
\ldots & \xrightarrow{M_k} & IVH_k & \xrightarrow{M_0} & IVH_{k+1} & \xrightarrow{M_1} & IHV_{k+1} & \xrightarrow{\cdots} \\
 & & = & & \neq & & = & \\
\ldots & \xrightarrow{M_k} & IVH_k & \xrightarrow{M_0'} & IVH_{k+1}' & \xrightarrow{M_1'} & IHV_{k+1} & \xrightarrow{\cdots}
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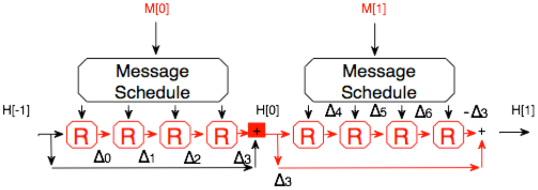

How to differentiate smartly

Differential notations

- M' denotes the collistion dual message of M
- $\Delta X = X' - X$ where $-$ denotes integer modular difference
- Applies to 32-bit components, e.g $\Delta IHV = (\Delta A, \Delta B, \Delta C, \Delta D)$
- $+2^{15} - 2^{8}$ means bit 15 flipped from 0 to 1 and bit 8 flipped from 1 to 0

Wang's collision path

$\delta m_4 = +2^{31}, \quad \delta m_{11} = +2^{15}, \quad \delta m_{14} = +2^{31}, \quad \delta m_i = 0, i \notin \{4, 11, 14\}$

t	ΔQ_t (BSDR of δQ_t)	δF_t	δw_t	δT_t	RC_t
$0-3$	$-$	$-$	$-$	$-$	$\cdot$
4	$-$	$-$	2^{31}	2^{31}	7
5	$+2^6 \ldots +2^{21}, -2^{22}$	$2^{11}+2^{19}$	$-$	$2^{11}+2^{19}$	12
6	$-2^6+2^{23}+2^{31}$	$-2^{10}-2^{14}$	$-$	$-2^{10}-2^{14}$	17
7	$+2^0 \ldots +2^4, -2^5, +2^6 \ldots +2^{10}$ $-2^{11}, -2^{23} \ldots -2^{25}, +2^{26} \ldots +2^{31}$	$-2^2+2^5+2^{10}$ $+2^{16}-2^{25}-2^{27}$	$-$	$-2^2+2^5+2^{10}$ $+2^{16}-2^{25}-2^{27}$	22
8	$+2^0+2^{15}-2^{16}+2^{17}$ $+2^{18}+2^{19}-2^{20}-2^{23}$	$2^6+2^8+2^{10}$ $+2^{16}-2^{24}+2^{31}$	$-$	$2^8+2^{10}+2^{16}$ $-2^{24}+2^{31}$	7
9	$-2^0+2^1+2^6+2^7-2^8-2^{31}$	$2^0+2^6-2^{20}$ $-2^{23}+2^{26}+2^{31}$	$-$	$2^0-2^{20}+2^{26}$	12
10	$-2^{12}+2^{13}+2^{31}$	$2^0+2^6+2^{13}-2^{23}$	$-$	$2^{13}-2^{27}$	17
11	$+2^{30}+2^{31}$	-2^0-2^8	2^{15}	$-2^8-2^{17}-2^{23}$	22
12	$+2^7-2^8, +2^{13} \ldots +2^{18}, -2^{19}+2^{31}$	$2^7+2^{17}+2^{31}$	$-$	$2^0+2^6+2^{17}$	7
13	$-2^{24}+2^{25}+2^{31}$	$-2^{13}+2^{31}$	$-$	-2^{12}	12
14	$+2^{31}$	$2^{18}+2^{31}$	2^{31}	$2^{18}-2^{30}$	17
15	$+2^3-2^{15}+2^{31}$	$2^{25}+2^{31}$	$-$	$-2^7-2^{13}+2^{25}$	22
16	$-2^{29}+2^{31}$	2^{31}	$-$	2^{24}	5
17	$+2^{31}$	2^{31}	$-$	$-$	9
18	$+2^{31}$	2^{31}	2^{15}	2^3	14
19	$+2^{17}+2^{31}$	2^{31}	$-$	-2^{29}	20
20	$+2^{31}$	2^{31}	$-$	$-$	5
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22	$+2^{31}$	2^{31}	$-$	2^{17}	14
23	$-$	$-$	2^{31}	$-$	20
24	$-$	2^{31}	$-$	$-$	5
25	$-$	$-$	2^{31}	$-$	9
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34	$-$	$-$	2^{15}	2^{15}	16
35	$\delta Q_{35} = 2^{31}$	2^{31}	2^{31}	$-$	23
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$51-59$	$\delta Q_t = 2^{31}$	2^{31}	$-$	$-$	$\cdot$
60	$\delta Q_{60} = 2^{31}$	$-$	2^{31}	$-$	6
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63	$\delta Q_{63} = 2^{31}+2^{25}$	2^{31}	$-$	$-$	21
64	$\delta Q_{64} = 2^{31}+2^{25}$	$\times$	$\times$	$\times$	$\times$

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-1	$-2^{25}+2^{26}+2^{31}$	$\times$	$\times$	$\times$	$\times$
0	$+2^{25}+2^{31}$	2^{31}	$-$	$-$	7
1	$+2^{25}+2^{31}$	2^{31}	$-$	2^{25}	12
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Sufficient conditions

- Once a valid path is found (*Wang did it "by hand", relying only on intuition !*), we must build a pair of blocks that follows it

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- Wang proposed a set of conditions derived by hand, she made mistakes

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- Set “chaining” differentials to prevent carries
- Derive conditions to control ΔF_i from first to last bit. If a contradiction arises, backtrack

An exemple

Sample path

- $F(X, Y, Z) = (X \wedge Y) \vee (\neg X \wedge Z)$
- We want $\Delta Q_{i-1} = \Delta Q_{i-2} = 0$, $\Delta Q_{i-3} = 2^5$, $\Delta F(Q_{i-1}, Q_{i-2}, Q_{i-3}) = 2^7$
- State equation :$\Delta F(Q_{i-1}, Q_{i-2}, Q_{i-3})$ is $R_j(\Delta Q_i - \Delta Q_{i-1}) - \Delta M_i - \Delta K_i - \Delta Q_{i-4}$
- $\Delta F(Q_{i-1}, Q_{i-2}, Q_{i-3}) = 2^7$ is impossible (no differential on 8th bit)
- So we add a bit differential in position 8 by expanding carry in $\Delta Q_{i-3} = 2^5$
- We add conditions $Q_{i-3}[1_6, 1_7, 0_8]$
- Now we have differentials in bit 6 and 7 that ΔF hasn't. Fortunately, F doesn't have differentials if bits 6 and 7 are set.
- Furthermore, $Q_{i-3}[0_8]$ yields $\Delta F(Q_{i-1}, Q_{i-2}, Q_{i-3}) = 2^7$, so we have our conditions.

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- Birthday attach on previous block can lead to *IHV* of the correct form
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- Only a few bits are changed in the two 512-bit collision blocks

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- Apply compression function while sufficient conditions are met.
- If a condition is not met in a relatively deep state of the function, try to patch the block you're building using message modification (precomputed modification that do not broke previous conditions for this path) or tunneling (backtrack to the first neutral bit and pray)

An impressive breach

Random-looking versus constructed

- Back in 1991, MD5 was designed using intuition rather than theory
- Using simple techniques and intuition, it was possible to find weak diffusion paths and exploit them
- Rivest has learned his lesson, SHA-3 candidate MD6 is proven secure against differential attacks
- Sequential approach replaced by parallel, tree-based scheme

Serious security implications

- Integrity bypassed in a minute
- Digital signature no longer to be trusted
- Fortunately complex enough to discourage real world attacks

References

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